

Calculus 1

Final Exam

November 3, 2023 (8:30 – 10:30)



Instructions

Please read the instructions!

1) Apply l'Hospital's Rule to find the limit $\lim_{x \rightarrow 0^+} \sin x \ln x$. Indicate the results (e.g. continuity, differentiation rules, limit laws) used in each step.

2) Consider the function $V(x) = ax^{-2} - bx^{-1}$, where $x > 0$ and a, b are positive constants. Compute the 2nd-degree Taylor polynomial for V centred around the point where it attains its minimum.

3) The *centroid* (point of balance) of a line segment is at its midpoint. The centroid of multiple line segments is obtained by taking the length-weighted average of centroids. Derive an integral formula for the coordinates (\bar{x}, \bar{y}) of the centroid of a curve $y = f(x)$, $a \leq x \leq b$ and use it to locate the centroid of the upper-semicircle of radius R centred at the origin. [You may assume that $f(x)$ has a continuous derivative over $[a, b]$.]

4) Evaluate the definite integral $\int_0^{2\sqrt{3}} \frac{x+2}{\sqrt{4+x^2}} dx$.

5) To account for the seasonal variation, we may let the relative growth rate vary in the logistic differential equation. For example, consider the equation

$$P'(t) = k(t)P(t) \left(1 - \frac{P(t)}{M}\right) \quad \text{with} \quad k(t) = 1 + \cos t, \quad P(0) = P_0 \in (0, M).$$

Solve the above initial value problem for $P(t)$. What is $\lim_{t \rightarrow \infty} P(t)$?

6) Solve the following initial value problem:

$$y''(x) + 2y'(x) + 5y(x) = 5 + 8e^x, \quad y(0) = 3, \quad y'(0) = 0.$$

[Hint: Use the method of undetermined coefficients.]

- write your name and student number on the top of each sheet of writing paper!
- use the writing (lined) and scratch (blank) paper provided, raise your hand if you need more paper
- start each question on a new page
- use a pen with black or blue ink
- do not use any kind of correcting fluid or tape
- any rough work should be crossed through neatly so it can be seen
- this exam is open-book, you may use the textbook or the lecture notes
- you may view your textbook or notes on your devices (tablet/laptop/etc.), but they have to be in airplane mode! **Phones are not allowed!**
- you are allowed to use a simple pocket calculator
- programmable/graphing calculators are not allowed
- your work should be clearly and logically structured
- explain your reasoning using words
- show all your calculations, an answer without any computation will not be rewarded
- each problem is worth 15 points
- upon completion¹ submit your worksheets at the front desk

¹At the end of the exam or after you finished, whichever is sooner.